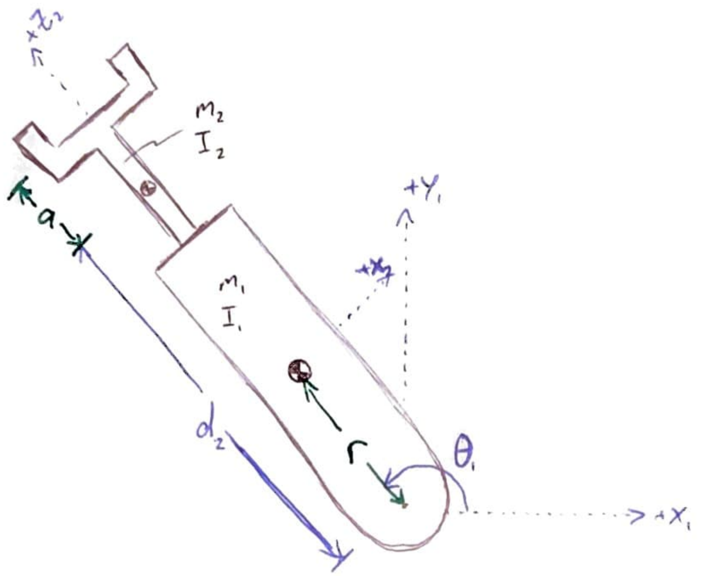

m_1, I_1 - mass & Inertia of link 1

m_2, I_2 - mass & Inertia of link 2

Center of Mass of link 1 : $\begin{bmatrix} r\cos\theta_1 \\ r\sin\theta_1 \end{bmatrix}$, $V_1 = \begin{bmatrix} -r\dot{\theta}_1 \sin\theta_1 \\ r\dot{\theta}_1 \cos\theta_1 \end{bmatrix}$ $\quad V_1^2 = V_1^T \cdot V_1 = r^2\dot{\theta}_1^2$

Center of Mass of link 2 : $\begin{bmatrix} d_2\cos\theta_1 \\ d_2\sin\theta_1 \end{bmatrix}$, $V_2 = \begin{bmatrix} \dot{d}_2\cos\theta_1 - d_2\dot{\theta}_1\sin\theta_1 \\ \dot{d}_2\sin\theta_1 + d_2\dot{\theta}_1\cos\theta_1 \end{bmatrix}$ $\quad V_2^2 = V_2^T \cdot V_2 = \dot{d}_2^2 + d_2^2\dot{\theta}_1^2$

$$\omega_1 = \omega_2 = \dot{\theta}_1$$

$$KE = \left(\tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}I_1\omega_1^2\right) + \left(\tfrac{1}{2}m_2 v_2^2 + \tfrac{1}{2}I_2\omega_2^2\right)$$

$$= \left[\tfrac{1}{2}m_1(r^2\dot{\theta}_1^2) + \tfrac{1}{2}I_1\dot{\theta}_1^2\right] + \left[\tfrac{1}{2}m_2(\dot{d}_2^2 + d_2^2\dot{\theta}_1^2) + \tfrac{1}{2}I_2\dot{\theta}_1\right]$$

$$PE = m_1 g r\sin\theta_1 + m_2 g d_2 \sin\theta_1$$

$$\Rightarrow L = KE - PE = \left(\tfrac{1}{2}\left[m_1 r^2\dot{\theta}_1^2 + I_1\dot{\theta}_1^2\right] + \tfrac{1}{2}\left[m_2(\dot{d}_2^2 + d_2^2\dot{\theta}_1^2) + I_2\dot{\theta}_1^2\right]\right) - \left(m_1 g r\sin\theta_1 + m_2 g d_2\sin\theta_1\right)$$

let $\gamma_1 = m_1 r^2 + I_1 + m_2 d^2 + I_2$ & $\gamma_2 = g(m_1 r_1 + m_2 d_2)$

Thus, $L = \frac{1}{2}\gamma_1\dot{\theta}_1^2 + \frac{1}{2}m_2\dot{d}_2^2 - \gamma_2 \sin\theta_2$

For θ_1: $2m_2 d_2 \dot{d}_2 \dot{\theta}_1 + \gamma_1 \ddot{\theta}_1 + \gamma_2 \cos\theta_1 = \tau_1$

For d_2: $m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 - g m_2 \sin\theta_1 = \tau_2$

Use Jacobian to find force-torque relationship

$$x = f(q) = [(d_2+a)\cos\theta_1,\ (d_2+a)\sin\theta_1]^T$$

$$\dot{x} = \dot{f}(q) = \frac{\partial f}{\partial q}\dot{q} = J(q)\dot{q} = \begin{bmatrix} -(d_2+a)\sin\theta_1 & \cos\theta_1 \\ (d_2+a)\cos\theta_1 & \sin\theta_1 \end{bmatrix}\dot{q}$$

$$\tau = J^T(q)F = \begin{bmatrix} -(d_2+a)\sin\theta_1 & (d_2+a)\cos\theta_1 \\ \cos\theta_1 & \sin\theta_1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\boxed{\tau = \begin{bmatrix} -3(d_2+a)\sin\theta_1 + 4(d_2+a)\cos\theta_1 \\ 3\cos\theta_1 + 4\sin\theta_1 \end{bmatrix}}$$