



$$T = \frac{1}{2} m \dot{x}^2$$

$$V = \frac{1}{2} k x^2$$

$$F = \frac{1}{2} b \dot{x}^2$$

$$q = x$$

$$\dot{q} = \dot{x}$$

$$\tau = 0 \in \text{no input force}$$

$$L = T - V = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + \frac{\partial F}{\partial \dot{q}} = \tau = 0$$

$$\Rightarrow \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{x}} = m \dot{x} \Rightarrow \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = m \ddot{x}$$

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial x} = -kx$$

$$\frac{\partial F}{\partial \dot{q}} = \frac{\partial F}{\partial \dot{x}} = b \dot{x}$$

$$\Rightarrow m \ddot{x} + kx + b \dot{x} = 0$$

$$\boxed{m \ddot{x} + b \dot{x} + kx = 0}$$